

Vertical Profiles of Wave-Coherent Momentum Flux and Velocity Variances in the Marine Atmospheric Boundary Layer

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ABSTRACT

The wave-coherent momentum flux and velocity variances are investigated using a theoretical model and open-ocean measurements. The spectrum-integrated wave-coherent (SIWC) momentum flux and velocity variances decay roughly exponentially with height. The exponential decay coefficients of the SIWC momentum flux and velocity variances decrease with increasing peak wavenumber. The phases of the wave-coherent horizontal (vertical) velocity fluctuations are approximately 180° (90°) under waves with wind-wave angle $|\alpha_1| < 90^\circ$. In general, the ratio of the SIWC momentum flux to the total momentum flux under swell conditions is higher than that under wind-wave conditions at the same height. At a height of 9.9 m, the SIWC vertical (horizontal) velocity variances can exceed 30% (10%) of the total vertical (horizontal) velocity variances at high wave ages. The impact of SIWC momentum flux on wind profiles is determined mainly by the surface SIWC momentum flux ratio, the decay coefficient of the SIWC momentum flux, and the sea surface roughness length, with the first two factors being dominant. The results of this study suggest a methodology for parameterizing the SIWC momentum flux and the total momentum flux over the ocean. These results are important for simulating the marine atmospheric boundary layer and should be used in model development.

1. Introduction

Air–sea interaction plays an essential role in weather forecasts and climate simulations. The existence of surface gravity waves makes the air–sea interface a very complex system that affects the turbulent boundary layer of the ocean and the atmosphere. Accordingly, surface waves can affect momentum flux, gas exchange, and heat exchange between the ocean and the atmosphere. Wind energy transported from the atmosphere to the ocean is one of the main drivers of ocean dynamics. The mechanical energy input from wind stress to the ocean is (Wang and Huang 2004a)

$$W_{\text{wind}} = TU_0 + \overline{T'_1 u'_0} + \overline{p' w'_0}, \quad (1)$$

where T and U_0 are the spatially averaged tangential stress and surface velocity at frequencies much lower than those of surface gravity waves, T'_1 and u'_0 are the high-frequency perturbed stress and velocity associated with surface waves, and p' and w'_0 are perturbations of the surface pressure and velocity components normal to the surface. The first term on the right-hand side of Eq. (1) represents the wind energy transported to geostrophic currents and the Ekman layer, estimated at approximately 3.9 TW globally (Wunsch 1998; Wang and Huang 2004a; Qiao and Huang 2012). The second and third terms on the right-hand side of Eq. (1) represent the wind energy transported from the wind to surface waves, estimated at approximately 60 TW globally (Wang and Huang 2004b; Qiao and Huang 2012). Many theories as well as open-ocean and

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laboratory experiments (Miles 1957; Hristov et al. 2003; Zavadsky and Shemer 2012) have addressed the wind-wave coupling in recent decades.

The critical layer theory (Miles 1957) is an inviscid wind-wave coupling model based on the concept of resonant wind-wave interaction. Energy transfer between a wave of phase speed c and the wind is directly related to the wind at the critical height z_c , where the mean wind speed is $U(z_c) = c$. The phases of wave-coherent (WC) horizontal (vertical) velocity fluctuations are approximately 180° (90°) below the critical height under wind-following wave conditions, and there is an abrupt change in the phase and amplitude of the WC velocity and pressure fluctuations at around the critical height (Hristov et al. 2003). Measurements from the open sea (Hristov et al. 2003; Grare et al. 2013) and laboratory experiments (Zavadsky and Shemer 2012) have indicated that the structure of airflow over waves agrees with results obtained using the critical layer theory.

Over the ocean, the total momentum flux τ can be divided into three parts: turbulent momentum flux τ_t , WC momentum flux τ_w , and viscous momentum flux τ_v . Viscous momentum flux is important only in the lowest millimeters of the atmospheric boundary layer (Smedman et al. 2003) and is neglected in this study. The ratio of the WC momentum flux to the total momentum flux decays with height, as indicated by measurements and numerical simulations (Sullivan et al. 2008; Höglström et al. 2015). Under growing wave conditions, the WC momentum flux is usually less than 10% of the total momentum flux at a height of approximately 6–10 m above the sea surface level (Grare et al. 2013; Hristov and Ruiz-Plancarte 2014). However, the ratio between the WC momentum flux and the total momentum flux can be much greater under fast wave (swell) and low wind conditions (Grare et al. 2013; Hristov and Ruiz-Plancarte 2014). At a height of 10.5 m, the WC momentum flux can be more than 20% of the total momentum flux under low friction velocity conditions, with $c_p/u_* > 60$, where c_p is the peak wave phase speed and u_* is the friction velocity (Grare et al. 2013).

Under swell and very light wind conditions, the total momentum flux can change its direction from downward to upward, that is, from ocean to atmosphere. Under upward total momentum flux conditions, the WC upward momentum flux is greater than the downward turbulent momentum flux. The WC upward momentum flux has been observed in the open ocean (Grachev and Fairall 2001; Smedman et al. 2009) and modeled in numerical simulations (Sullivan et al. 2008). A low-level maximum wind occurs in the near surface under conditions when the total momentum

flux is upward (Smedman et al. 2009; Semedo et al. 2009; Jiang et al. 2016). However, common atmospheric models cannot capture the wave-induced low-level wind jet, for example, Weather Research and Forecasting (WRF) Model and Rossby Center Regional Atmospheric Model (RCA). To capture wave-induced low-level wind jets, several surface wind profile models (Hanley and Belcher 2008; Semedo et al. 2009; Song et al. 2015) have been proposed to introduce the WC momentum flux into common surface wind models.

Parameterizing the wind stress (i.e., momentum flux) over the ocean is important for the development of atmospheric and oceanic models. In most parameterizations, the surface momentum flux is calculated using bulk formulas usually based on Monin–Obukhov similarity theory (MOST). However, the scatter of the wind stress between model results and measurements is nonnegligible, especially under swell-dominated wave conditions ($c_p/U_{10} > 1.2$, where U_{10} is the mean wind speed at 10 m above the sea surface level). To improve the performance of wind stress parameterizations, many efforts have focused on parameterizing the surface roughness length through a wave state-dependent Charnock coefficient (wave age, wave steepness, etc.; e.g., Drennan et al. 2005). Höglström et al. (2015) recently parameterized the total momentum flux under swell conditions using two separate terms, that is, the peak swell-contributed momentum flux and the residual momentum flux, in an approach differing from traditional parameterizations. The drag coefficient of the residual momentum flux is parameterized as a function of U_{10} . The peak swell-contributed momentum flux is parameterized using wave parameters. Höglström et al. (2015) found that the peak swell-contributed momentum flux decayed approximately exponentially with height, that is, $dP(z) = dP_0 e^{-A_{uw}k_p z}$, where k_p is the peak wavenumber, dP_0 is the peak swell-supported momentum flux at the surface, A_{uw} is a decay coefficient, and z is the height above the mean sea surface level. Höglström et al. (2015) used a straightforward graphical method to identify the peak–swell-contributed momentum flux and found the decay coefficient A_{uw} to be in the 1–2.1 range. To calculate the energy transfer between wind and waves, Kahma et al. (2016) extrapolated WC pressure fluctuations at the water surface from a given measurement height by assuming that WC pressure fluctuations follow the exponential decay equation, that is, e^{-kz} , where k is the wavenumber.

It is impossible to measure the WC momentum flux and pressure directly at the air–sea boundary because of the possible impact of sea spray and bubbles on instruments. The assumption that the WC momentum flux

decays exponentially with height is commonly used in estimating the surface WC momentum flux and surface pressure fluctuations (Semedo et al. 2009; Höglström et al. 2015). However, the decay coefficient A_{uw} varies between studies, having values of $A_{uw} = 2$ (Semedo et al. 2009; Song et al. 2015), $A_{uw} = 10$ (Hanley and Belcher 2008), $A_{uw} = 1.4$ (Höglström et al. 2015), as well as $A_{uw} = 5$ for short waves (Makin and Mastenbroek 1996). It is therefore possible that some other factors could affect the decay coefficient of the WC momentum flux.

The logarithmic wind profile has been widely used in the surface layer of atmospheric models. Over the ocean, wave influences on the wind profile have been observed from measurements and modeled in numerical simulations (Höglström et al. 2009; Sullivan et al. 2008). Höglström et al. (2009) calculated the pressure transport term (a wave-related term) in turbulent kinetic energy (TKE) budgets. The profile of the wave-induced pressure transport term is important for the departure of the wind profile from the logarithmic wind profile. After taking out the wave influence term, the wind profile is approximately logarithmic. In surface wind models (e.g., Semedo et al. 2009), the impact of the WC momentum flux profile is another important factor affecting the logarithmic wind profile.

The exponential decay coefficient of WC momentum flux is an important factor when parameterizing the surface WC momentum flux and estimating the wave impact on the wind profile. However, to our knowledge, few studies have investigated the profile of the WC momentum flux and velocity variances. In this study, we accordingly address the following questions:

- Can the WC momentum flux and velocity variances be expressed as exponentially decaying functions of height, as in the equations

$$\overline{u\tilde{w}}(z) = \overline{u\tilde{w}}_0 e^{-A_{uw}k_p z}, \quad (2)$$

$$\overline{u^2}(z) = \overline{u^2}_0 e^{-A_u k_p z}, \quad \text{and} \quad (3)$$

$$\overline{\tilde{w}^2}(z) = \overline{\tilde{w}^2}_0 e^{-A_w k_p z}, \quad (4)$$

where $\overline{u\tilde{w}}_0$, $\overline{u^2}_0$, and $\overline{\tilde{w}^2}_0$ are the WC momentum flux; WC horizontal and vertical velocity variances at the sea surface, respectively; and A_u and A_w are the decay coefficients of the WC horizontal and vertical velocity variances, respectively?

- If the WC momentum flux and velocity variances decay approximately exponentially with height, on which parameters do the exponential decay coefficients, that is, A_{uw} , A_u , and A_w , depend? Or can they be treated as constants, as is done in some studies (e.g., Semedo et al. 2009)?

- How is the logarithmic wind profile sensitive to the WC momentum flux?

These three questions are investigated here based on a theoretical model and open-ocean measurements. The rest of the paper is organized as follows: The theoretical model and its results for the profiles of spectrum-integrated wave-coherent (SIWC) momentum flux and velocity variances are presented in section 2. The measurements used in this study and the results for the profiles of SIWC momentum flux and velocity variances are presented in section 3. The wave impact on the logarithmic wind profile is discussed in section 4. Sections 5 and 6 present the discussion and concluding remarks, respectively.

2. Theoretical model

a. Model

For a stationary, horizontally homogeneous, and neutrally stratified atmospheric boundary layer with velocity $U + u, v, w$, the airflow fluctuation velocity over waves can be expressed as $u = \tilde{u} + u'$, $v = \tilde{v} + v'$, and $w = \tilde{w} + w'$, where u', v', w' are the turbulent fluctuating velocities and $\tilde{u}, \tilde{v}, \tilde{w}$ are the WC fluctuating velocities. For wind-aligned waves, $\tilde{v}$ is zero. Following the study of Hristov and Ruiz-Plancarte (2014), under neutral stratification conditions with small slope waves [$\eta = a e^{ik(x-ct)}$], the wave-correlated fluctuations can be described using the Rayleigh equation:

$$\phi'' - (1 + \chi''/\chi)\phi = 0, \quad (5)$$

where ϕ is a transfer function for the airflow response Ψ (i.e., a streamfunction) to the wave forcing η , and χ is a dimensionless variable, as in $\chi = (U - c)/(u_* \kappa)$ (where U is the mean wind speed, c is the wave phase speed, u_* is the friction velocity, and κ is the von Kármán constant). Details of the equation refer to Hristov and Ruiz-Plancarte (2014). The WC velocities for certain wave modes can be expressed as

$$\tilde{u} = \frac{u_*}{\kappa} \frac{d\phi}{d\xi} k \eta, \quad \text{and} \quad (6)$$

$$\tilde{w} = i \frac{u_*}{\kappa} \phi k \eta, \quad (7)$$

assuming the wave spectrum aligns with the wind in the form $F(\omega) \propto \omega^{-\beta}$ and $\beta > 1$. The term β is usually 5 (the Phillips spectrum) or 4 in the case of the Donelan–Hamilton–Hui (DHH) spectrum (Donelan et al. 1985). The wave slope spectrum $k^2 F(\chi_0)$ can be expressed as follows (Hristov and Ruiz-Plancarte 2014):

$$k^2 F(\chi_0) = \begin{cases} (\beta - 1)(\sigma_\eta k_p)^2 \chi_0^{\beta-6} / \chi_{0p}^{\beta-6} & \text{for } \chi_{0p} \leq \chi_0 < 0 \\ 0 & \text{for } \chi_0 < \chi_{0p} \end{cases}, \quad (8)$$

where $\chi_0 = \chi(z_0) = -\kappa c / u_*$ (z_0 is the surface roughness length), $\chi_{0p} = -\kappa g / (\omega_p u_*)$ is the wave age at the spectral peak, $\sigma_\eta k_p$ is the wave slope, and $\sigma_\eta^2 = \int_{\omega_p}^\infty F(\omega) d\omega$ is the variance. Accordingly, the WC velocity variances can be expressed as

$$\tilde{u}^2(\chi_0) = \frac{u_*^2}{\kappa^2} (\beta - 1)(\sigma_\eta k_p)^2 \frac{\chi_0^{\beta-6}}{\chi_{0p}^{\beta-6}} |\phi'|^2, \quad \text{and} \quad (9)$$

$$\tilde{w}^2(\chi_0) = \frac{u_*^2}{\kappa^2} (\beta - 1)(\sigma_\eta k_p)^2 \frac{\chi_0^{\beta-6}}{\chi_{0p}^{\beta-5}} |\phi'|^2. \quad (10)$$

Therefore, the profiles of the SIWC horizontal and vertical velocity variances are

$$\overline{\tilde{u}^2(z)} = \int_{\chi_{0p}}^0 \tilde{u}^2(z, \chi_0) d\chi_0, \quad \text{and} \quad (11)$$

$$\overline{\tilde{w}^2(z)} = \int_{\chi_{0p}}^0 \tilde{w}^2(z, \chi_0) d\chi_0; \quad (12)$$

the SIWC momentum flux is expressed as (Hristov and Ruiz-Plancarte 2014)

$$\overline{\tilde{u}\tilde{w}(z)} = \int_{-\infty}^{-\log(z/z_0)} \tilde{u}\tilde{w}(z; \chi_0) d\chi_0; \quad (13)$$

and $\tilde{u}\tilde{w}(z; \chi_0)$ is expressed as

$$\tilde{u}\tilde{w}(z; \chi_0) = \begin{cases} \frac{1-\beta}{\kappa^2} u_*^2 (\sigma_\eta k_p)^2 \left(\frac{\chi_0}{\chi_{0p}} \right)^{\beta-5} \text{Im}[\phi'_0(\chi_0)] & \text{for } z_0 \leq z \leq z_c, \\ 0 & \text{for } z_c < z \end{cases}, \quad (14)$$

where $\phi'_0(\chi_0) = [d\phi(\xi; \chi_0)/d\xi]|_{\xi=\xi_0}$, $\xi = kz$, and $\xi_0 = kz_0$.

b. Vertical profiles

To determine the WC momentum flux and velocity variances, the logarithmic wind profile is used to illustrate the results:

$$U(z) = \frac{u_*}{\kappa} \ln(z/z_0). \quad (15)$$

The surface roughness length can be calculated using the Charnock relationship $z_0 = \Omega(u_*/\kappa)^2/g$, where Ω is the wind profile parameter (Miles 1957), and g is the gravitational acceleration. Many studies have demonstrated that the Charnock coefficient, that is, $\alpha = \Omega/\kappa^2$, is related to wave states (e.g., Janssen 1989, 1991; Drennan et al. 2005), though the precise relationship differs between studies (e.g., Guan and Xie 2004). To achieve the aim of the sensitivity test, several values of Ω were used to test its influence on simulation results.

Figure 1 shows the profiles of the normalized SIWC momentum flux and velocity variances for an example case with $\sigma_\eta k_p = 0.026$, $u_* = 0.4 \text{ m s}^{-1}$, $\Omega = 0.01$, $\beta = 4$, and $c_p/u_* = 30.6$ using the theoretical model. The black lines in Fig. 1 are the results calculated from the theoretical model (section 2a). With increasing height, the SIWC momentum flux (Fig. 1c) and velocity variances (Figs. 1a,b) decrease rapidly. The SIWC velocity

variances decay more slowly with height than does the SIWC momentum flux. Compared with the SIWC horizontal velocity variances, the SIWC vertical velocity variance decays more slowly. The red lines in Fig. 1 are the exponential fit lines in the form of Eqs. (2), (3), and (4) (using the theoretical model result data between 2 and 15 m). One can see that the profiles of the normalized $\overline{\tilde{u}^2}$, $\overline{\tilde{w}^2}$, and $\overline{\tilde{u}\tilde{w}}$ agree well with the regression lines (the regression coefficient $R^2 > 0.94$ in all cases). Very near the surface, the difference between the normalized theoretical model results and the regression lines is somewhat larger. However, if we look at the absolute values of the WC velocity variances and momentum flux, the deviation of the numerical simulation results from the regression lines is very small. According to the theoretical model, it is therefore reasonable to parameterize the WC momentum flux and velocity variances using exponential decay functions.

According to the theoretical model [i.e., Eqs. (11), (12), and (13)], the SIWC velocity variances and momentum flux are determined by many parameters, such as k_p , β , and z_0 (z_0 is determined by Ω and u_*). Based on Eqs. (9), (10), and (14), one can know that $\sigma_\eta k_p$ can only affect the magnitude of the WC variances and momentum but not the shape of their profiles. Thus, the impact of $\sigma_\eta k_p$ on the results is not discussed in this study. To investigate whether the decay coefficients in

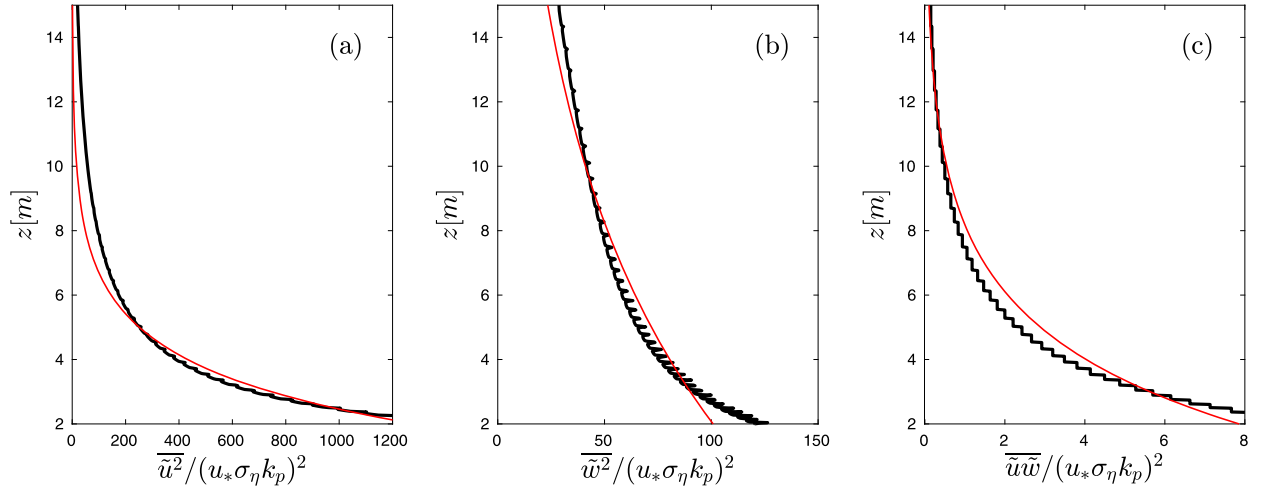


FIG. 1. The profiles of the normalized SIWC horizontal and vertical velocity variances (a) $\bar{u}^2$ and (b) $\bar{w}^2$ and (c) the normalized SIWC momentum flux $\bar{u}\bar{w}$. The black lines are the model simulation results, and the red lines are the exponential decay fit lines.

Eqs. (2), (3), and (4) should be treated as constants, as suggested in many studies (Semedo et al. 2009; Höglström et al. 2015), more sensitivity cases (with varying k_p , β , and Ω) are simulated based on the case shown in Fig. 1. The range of k_p is 0.037–0.147 m^{-1} and of β is 4–5; three values of Ω , that is, 0.003, 0.01, and 0.02, are used. The decay coefficients calculated according to Eqs. (2), (3), and (4) are shown in Figs. 2 and 3. The regression coefficients for all sensitivity cases are higher than 0.93 ($R^2 > 0.93$).

Figure 2 shows the change in decay coefficients with peak wavenumber k_p and with Ω . With increasing k_p , the decay coefficients (i.e., A_u , A_w , and A_{uw}) decrease, meaning that the SIWC velocity variances and momentum flux decay more slowly for waves with

higher peak wavenumbers. The decay coefficients A_{uw} and A_u are more sensitive to the peak wavenumber than is A_w . The solid lines in Fig. 2 are the exponential regression lines for the decay coefficients as they change with k_p . The decay coefficients decay approximately exponentially with k_p . The impact of Ω on the decay coefficients is shown in Fig. 2 by lines of different colors. The changing of Ω does not have a significant impact on the decay coefficient A_{uw} . However, the decay coefficients A_u and A_w increase with increasing Ω , and A_w is more sensitive to Ω than are A_{uw} and A_u .

The wave spectrum exponent β is commonly observed in the range of $4 \leq \beta \leq 5$ for the equilibrium wave spectrum. The impact of β on the decay coefficients is shown in Fig. 3. With increasing β , the decay

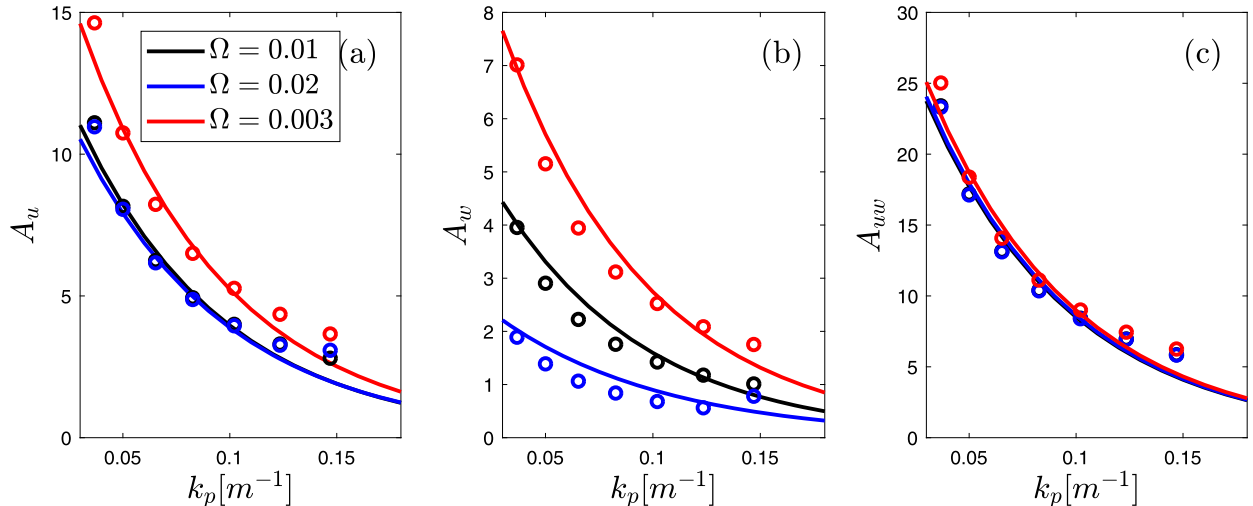


FIG. 2. The change in decay coefficients (a) A_u , (b) A_w , and (c) A_{uw} with peak wavenumber k_p . The black, blue, and red open circles are the results for the cases $\Omega = 0.01$, 0.02, and 0.003, respectively; the solid lines are exponential decay fit lines.

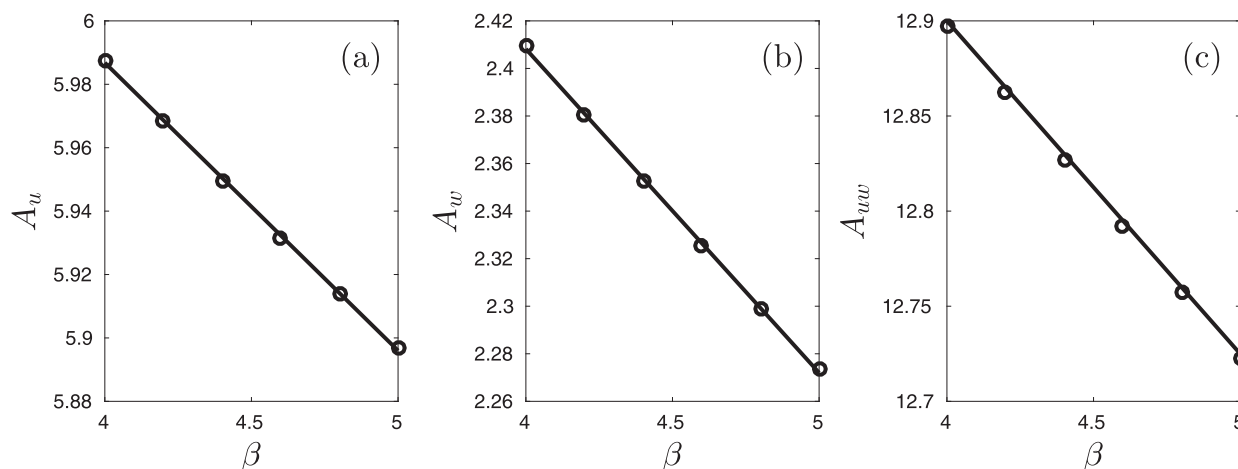


FIG. 3. The change in decay coefficients (a) A_u , (b) A_w , and (c) A_{uw} with the wave spectrum exponent parameter β . The black open circles are the results for the cases with $u_* = 0.2$, $k_p = 0.065$, and $\Omega = 0.01$; the solid lines are linear fit lines.

coefficients, that is, A_u , A_w , and A_{uw} , decrease approximately linearly. Compared with k_p , the wave spectrum exponent has a smaller impact on the decay coefficients.

The impact of u_* on the decay coefficients was also tested, though it displayed no significant impact on them. Compared with the wave spectrum exponent β , the peak wavenumber k_p and Ω have greater impacts on the decay coefficients A_u and A_w . In the case of A_{uw} , the peak wavenumber is the dominant factor given the logarithmic wind profile used in the theoretical model.

3. Experimental measurement results

a. Data and methods

1) DATA

The measurements from the Rough Evaporation Duct (RED) experiment were used in this study to investigate the profile of SIWC momentum flux and variances. The experiment was conducted from late August to mid-September 2001, and the data were collected from the research platform (R/P) *Floating Instrument Platform (FLIP)* approximately 10 km off the northeastern coast of Oahu. The water depth was approximately 370 m. The keel of R/P *FLIP* was aligned with the prevailing northeasterly trade wind direction, and its port boom extended approximately 17 m from the hull in a northerly direction. Three-dimensional (3D) mean and turbulent wind and temperature were measured at heights of 5.1, 6.9, 9.9, 13.8 m (by Campbell CSAT3 sonic anemometers), and 16.8 m (by a Gill Sonic sensor) above the mean sea surface level on the vertical mast. A cup and vane were installed at a height of 19.8 m to measure the mean wind speed and direction. The mean specific humidity and temperature were

measured using EdgeTech dewpointers and Hart thermistors at the same five heights at which the turbulent wind and temperature were measured. In the present study, the 50-Hz velocity components measured at heights of 5.1, 6.9, 9.9, 13.8, and 16.8 m are used. The 50-Hz surface elevation data from wave staffs (using Scripps Institution of Oceanography wave wire) are used for wave information. For details on the experiment and data, refer to [Anderson et al. \(2004\)](#) and [Högström et al. \(2013\)](#).

The time series of the RED data is shown in [Fig. 4](#). The wind speed at 9.9 m, that is, $U_{9.9}$, was in the 2–12 m s^{-1} range. The significant wave height was 0.4–2 m during the experimental period. In the experimental period, the peak wave direction was aligned with wind direction, or the wind-wave angle is smaller than 90° , and the surface waves were dominated by swell waves ($c_p/U_{9.9} > 1.2$). The spectrogram of the surface displacement is shown in [Fig. 4](#). Most of the wave energy was found in the wave modes with a phase speed in the 5–20 m s^{-1} range. The difference between the air temperature and sea surface temperature was -0.5° – 2°C in the experimental period, meaning that the conditions during the experiment were slightly unstable, that is, the z/L is in the range -0.2 to 0 , where L is the Obukhov length. The momentum fluxes calculated from the five levels are roughly the same; the variation is typically less than $\pm 5\%$ for individual 30-min runs (see [Fig. 2c](#) in [Högström et al. 2013](#)).

2) METHODS

Estimating the SIWC momentum flux from field measurements is vital when investigating the profile of the SIWC momentum flux. Several methods have been used to estimate the WC momentum flux, such as phase

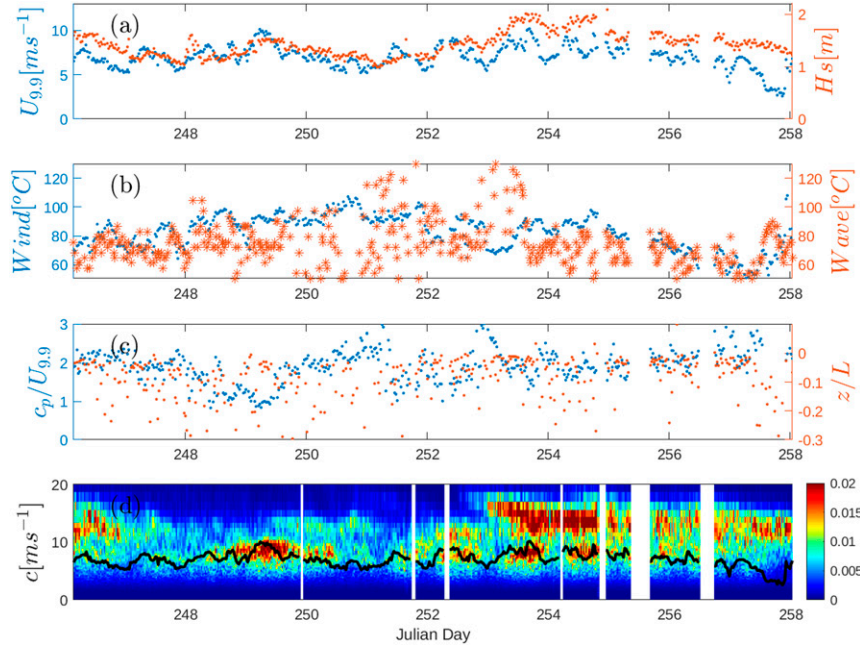


FIG. 4. The time series of (a) mean wind speed at 9.9 m (blue) and significant wave height H_s (red); (b) the wind (blue) and wave directions (red; °); (c) the wave age $c_p/U_{9.9}$ (blue), where c_p is the peak wave speed, and atmospheric stability condition z/L (red), where L is the Obukhov length (negative value of z/L means unstable atmospheric conditions); and (d) spectrogram of the surface displacement, that is, $S_{\eta\eta}(c)$. The black line in (d) is $U_{9.9}$.

averaging (Hristov et al. 1998), linear transformation (Veron et al. 2008; Grare et al. 2013), and the orthogonal projection of the wind onto the Hilbert space to estimate the WC signal (Hristov et al. 2003; Hristov and Ruiz-Plancarte 2014). Before we start to analyze the SIWC momentum flux and variances, the results of two methods are compared with each other. The methods used in the comparison are summarized as follows:

- Method 1

The WC velocity can be calculated by projecting the measured velocity onto the Hilbert space of the WC signals (Hristov et al. 1998, 2003), which can be expressed as

$$\tilde{u}(t) = \Sigma \left[\frac{\langle u, \eta_n \rangle}{\eta_n^2} \eta_n(t) + \frac{\langle u, \hat{\eta}_n \rangle}{\hat{\eta}_n^2} \hat{\eta}_n(t) \right], \quad (16)$$

where $\langle u, \eta \rangle$ is a time-averaged product of u and η and $\hat{\eta}$ is the Hilbert transform of η . In this method, the wave signal $\eta(t)$ is split into a set of narrowband components $\{\eta_n(t)\}$. Accordingly, the SIWC momentum flux $\tilde{u}\tilde{w}$ and velocity variances $\tilde{u}^2$ and $\tilde{w}^2$ can be calculated using the WC velocities from Eq. (16).

- Method 2

Veron et al. (2008) and Grare et al. (2013) calculated the SIWC momentum flux using

$$\overline{\tilde{u}\tilde{w}} = \int (S_{\tilde{u}\tilde{u}} S_{\tilde{w}\tilde{w}})^{0.5} \cos(\phi_{\tilde{u}} - \phi_{\tilde{w}}) df, \quad (17)$$

where the spectral density of WC fluctuations $\tilde{w}$ and $\tilde{u}$ are $S_{\tilde{w}\tilde{w}} = S_{ww} \gamma_{\eta w}^2$ and $S_{\tilde{u}\tilde{u}} = S_{uu} \gamma_{\eta u}^2$, respectively; the squared coherence $\gamma_{\eta u_i}^2$ between the velocity u_i and the surface displacement η is

$$\gamma_{\eta u_i}^2 = \frac{|S_{\eta u_i}|^2}{S_{\eta\eta} S_{u_i u_i}}. \quad (18)$$

The phase shifts of the surface displacement and the WC horizontal and vertical velocities are represented by $\phi_{\eta\tilde{u}}$ and $\phi_{\eta\tilde{w}}$, respectively.

The SIWC momentum flux is calculated using 30-min-segment high-frequency data for each flux estimation. The auto and cross spectra used in method 2 are computed using 6000-point fast Fourier transforms (FFTs) in 30-min segments. To reduce noise influences, turbulences slower than 30 s are filtered out in method 1. Figure 5 shows the SIWC momentum flux calculated using the two above methods. The results of the two methods agree relatively well with each other, with some scatter, but no systematic differences between the results of the two methods. The results calculated using

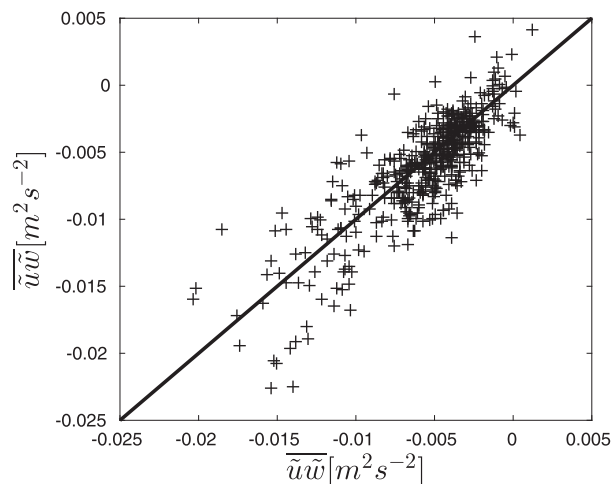


FIG. 5. Comparison of the SIWC momentum flux at 9.9 m calculated using two different methods: the x axis presents the results of method 1 and the y axis presents the results of method 2; the black line is the 1:1 line.

method 2 differ somewhat if different numbers of data points are used to do the FFTs; to prevent this from affecting the results, the results of method 1 are used in the following sections (though the following results do not differ significantly if method 2 is used).

The fraction of the SIWC velocity variances changing with $c_p/U_{9.9}$ is shown in Figs. 6a and 6b. The SIWC horizontal velocity variance $\tilde{u}^2$ is approximately 1%–10% of the total horizontal velocity variance u^2 at the height of 9.9 m in most cases. The SIWC vertical velocity variance $\tilde{w}^2$ is approximately 5%–30% of the

total vertical velocity variance w^2 . In general, the SIWC velocity variances take a larger portion of the total velocity variances under low wind and high wave age conditions. The ratio of the SIWC vertical velocity variance to the total vertical velocity variance is more sensitive to the wind speed and $c_p/U_{9.9}$ than is $\tilde{u}^2/u^2$. The ratio of the SIWC velocity variance to the total velocity variance, that is, $\tilde{u}^2/u^2$ and $\tilde{w}^2(w)/w^2(z)$, decreases with height. The magnitude of $\tilde{w}^2/w^2$ decays faster with height than that of $\tilde{u}^2/u^2$ and $\tilde{u}\tilde{w}/\overline{uw}$ (figure not shown).

The magnitude of the SIWC momentum flux at 9.9 m was less than $0.01 m^2 s^{-2}$ for the most time of the experimental period (see Fig. 5). Figure 6c shows the ratio between the SIWC momentum flux and the total momentum flux at 9.9 m. In most cases, the SIWC momentum flux is 5%–25% of the total momentum flux. In some cases, mainly under low wind and high wave age ($c_p/U_{9.9}$) conditions, the SIWC momentum flux exceeds 35% of the total momentum flux. Under low winds, the wind shear is small, leading to a small turbulent momentum flux. The SIWC momentum flux therefore represents a larger portion of the total momentum flux under low wind with high wave age (swell wave) conditions than that under wind-wave conditions [cf. the ratio of the SIWC momentum flux to the total momentum flux under swell conditions in this study and the results from Grare et al. (2013) and Hristov and Ruiz-Plancarte (2014) under wind-wave conditions]. In a few cases, mainly under very low winds, the SIWC momentum flux is upward, that is, from ocean to atmosphere.

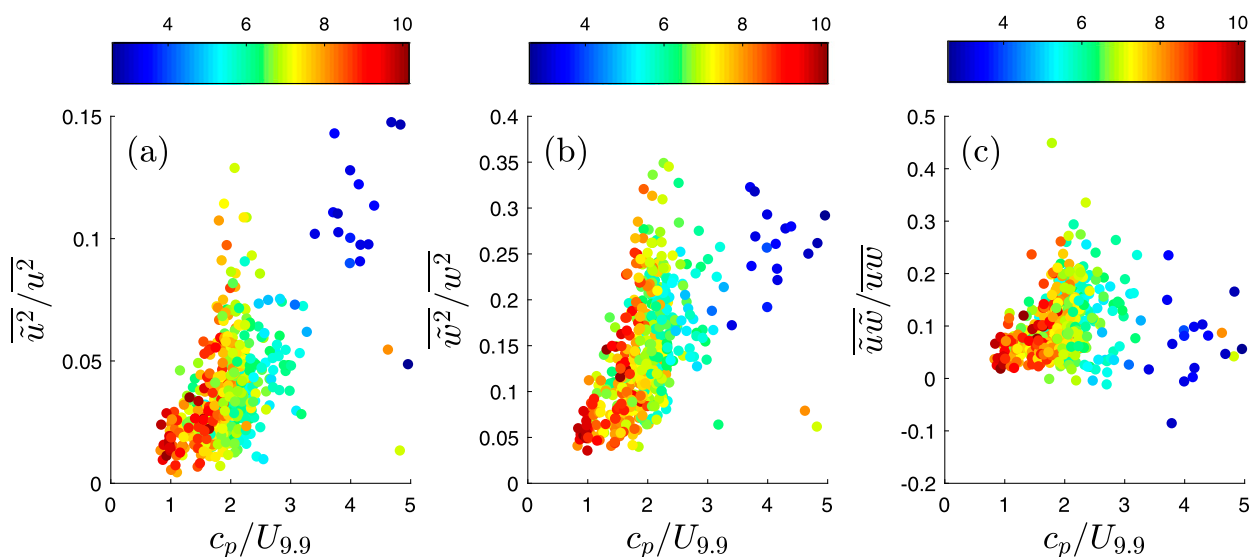


FIG. 6. The ratios of the SIWC momentum flux and variances to the total momentum flux and variances at 9.9 m: (a) $\tilde{u}^2/u^2$, (b) $\tilde{w}^2/w^2$, and (c) $\tilde{u}\tilde{w}/\overline{uw}$. The color bar represents $U_{9.9}$.

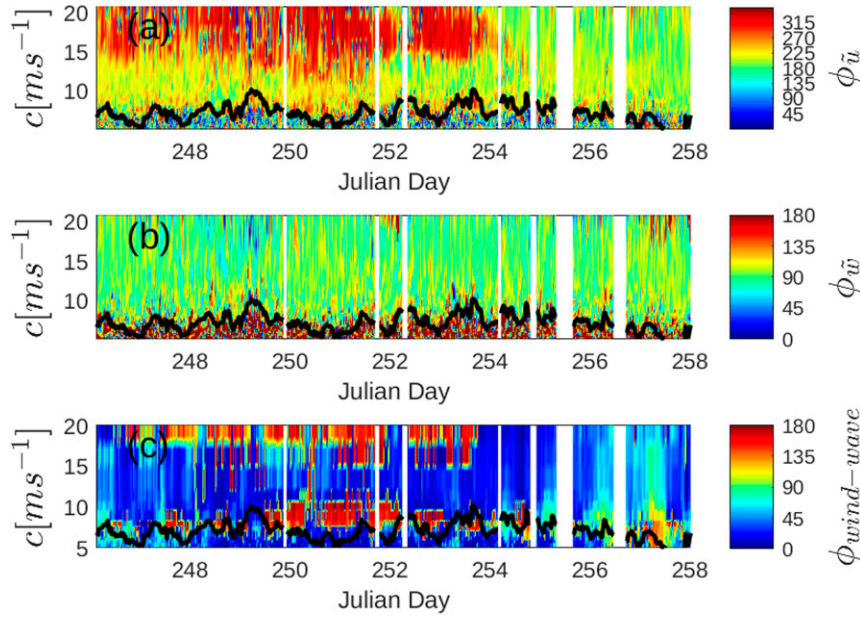


FIG. 7. The phase ($^{\circ}$) of (a) $\tilde{u}$ and (b) $\tilde{w}$ at height 9.9 m. (c) The wave angle difference between the wind and the peak wave direction for each wave mode; the black lines represent $U_{9.9}$.

b. Phase shift

The critical layer model (section 2) has been proved by measurements made under wind-wave conditions (Hristov et al. 2003; Grare et al. 2013). During the RED experiment, wind-following swell or wind-swell angle $|\alpha_1| < 90^{\circ}$ waves dominated the wave conditions. Here, the turbulence structure over swell waves is investigated using RED data, comparing these with the critical layer model results. The transfer functions between WC horizontal and vertical velocities and η are expressed as $T_{\tilde{u}}(c) = S_{\eta\tilde{u}}/S_{\eta\eta}$ and $T_{\tilde{w}}(c) = S_{\eta\tilde{w}}/S_{\eta\eta}$, respectively. The phases of $\tilde{u}$ and $\tilde{w}$ are $\phi_{\tilde{u}} = \arg(T_{\tilde{u}})$ and $\phi_{\tilde{w}} = \arg(T_{\tilde{w}})$, respectively.

The $\phi_{\tilde{u}}$ and $\phi_{\tilde{w}}$ for the RED data at 9.9 m are shown in Figs. 7a and 7b. The black lines in the figures indicate the wind speed at 9.9 m, with above (below) the black line indicating below (above) the critical layer. The theoretical model has shown that $\phi_{\tilde{u}}$ ($\phi_{\tilde{w}}$) are approximately 180° (90°) under wind-following wave conditions (Hristov et al. 2003). In general, the results of RED data agree with the theoretical model results. However, on days 246–254 for very fast wave modes, the phase of $\tilde{u}$ is approximately 360° (0°) below the critical layer. One possible reason is that the wave direction in those wave modes opposes the wind direction [see Fig. 7c; the directional wave spectrum is calculated using the method in Kundu and Cohen (2002)]. On days 246–254, there is more than one swell system. One example of 2D wave spectra is shown in Fig. 8. One can see that there is

another swell system in high-phase speed wave modes. The direction of the weak swell opposes that of the main swell system. Another possible reason is that the signal-to-noise ratio is low for low wave energy modes. The phase of $\tilde{w}$ is approximately 90° (see Fig. 7b) below the critical height, and there is a sudden change around the critical layer height, which agrees with the theoretical model results (Hristov et al. 2003).

In general, the phase structure of wave-coherent fluctuations under swell conditions agrees with the

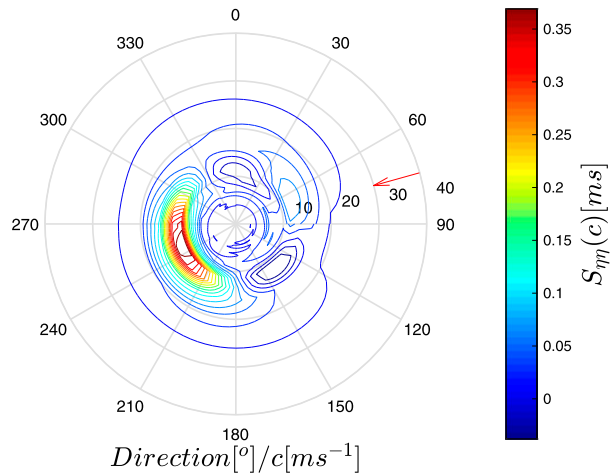


FIG. 8. The 2D wave spectrum on day 246.5417; the red arrow indicates the wind direction, and the concentric circles represent the wave speed c .

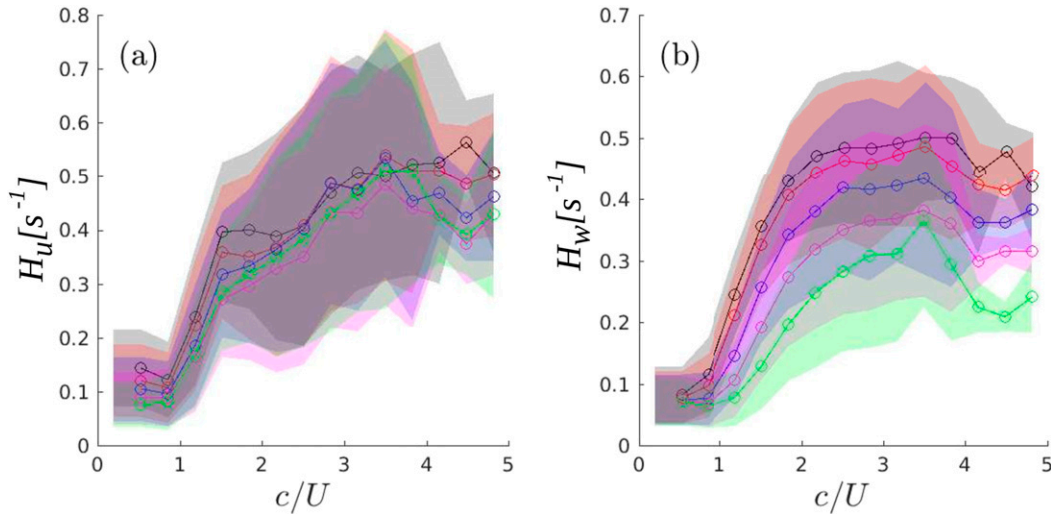


FIG. 9. The normalized amplitudes of the wave-induced velocities (a) H_u and (b) H_w (s^{-1}) at different levels. The black, red, blue, magenta, and green lines indicate the measurements at heights of 5.1, 6.9, 9.9, 13.8, and 16.8 m, respectively; the shaded areas represent the standard deviations.

results under the wind-wave conditions and the results of the critical layer model.

c. Normalized amplitudes of transfer functions

The normalized amplitudes of transfer functions for $\tilde{u}$ and $\tilde{w}$ are $H_u = |T_{\tilde{u}}|$ and $H_w = |T_{\tilde{w}}|$, respectively. In this section, the normalized amplitudes of transfer functions changing with height and wave age are investigated using the RED data.

The normalized amplitudes of transfer functions H_u and H_w for different wave modes at different heights are shown in Fig. 9. The solid lines are the bin average of H_u and H_w at different spectral wave ages, that is, c/U , while their standard deviations are shown by the shaded areas. Although the wind-wave cases were not common in the RED experiment, the sudden change of H_u and H_w at approximately $c/U = 1$ is significant, as it is around the critical layer height. The change of H_u and H_w around the critical height agrees with the results of measurements (Grare et al. 2013) and of the theoretical model (Hristov et al. 2003). The values of H_u and H_w decrease with increasing c/U in the range $c/U < 1$ [the RED measurements display large scatter because of the limited number of measurements in this range; however, the data used by Grare et al. (2013) show this trend clearly, as is evident in their Fig. 8] and increase with increasing c/U in the range $c/U > 1$. The minimum values of H_u and H_w occur at approximately $c/U = 1$. In the study of Grare et al. (2013), the maximum value of c/U is approximately 3. In the RED data, the values of H_u and H_w stop increasing when c/U exceeds 3–4. In the case of H_w , the value even starts to decrease with increasing c/U when c/U exceeds 4.

In general, H_u and H_w decrease with increasing height (though there is some scatter in H_u at a height of 16.8 m). Compared with H_u , H_w decays more quickly with height, meaning that the WC signal in the vertical direction decays more quickly than that in the horizontal direction.

d. Decay coefficient

Half-hour samples (at day 247.083) of the WC velocity spectrum for $S_{\tilde{u}\tilde{u}}$ and $S_{\tilde{w}\tilde{w}}$ at different heights are shown in Figs. 10a and 10c. The real part of the complex co-spectrum between $\tilde{u}$ and $\tilde{w}$, that is, $S_{\tilde{u}\tilde{w}}$, for the same samples is shown in Fig. 10e. The WC energy exists mainly in high wave energy modes at phase speeds of $10\text{--}20 \text{ m s}^{-1}$ at the height of the anemometers. The WC energy decays with height as shown in Figs. 10a, 10c, and 10e. This agrees with the results for the amplitude of the transfer function, which decreases with height (see Fig. 9). The SIWC velocity variances and momentum flux, shown in Figs. 10b, 10d, and 10f, decay approximately exponentially with height. The dashed lines are the exponential fit lines to the measurements, which agree well with the measurement results.

The theoretical model shows that the decay coefficient is related to the wave spectrum exponent β , Charnock coefficient ($\alpha = \Omega/\kappa^2$), and peak wavenumber k_p . The decay coefficients are more sensitive to k_p and Ω than to the other parameters, as indicated by the theoretical model results. The wind profile parameter Ω is an experimental parameter. There are many studies stating that the Charnock number is a function of wave state, such as wave age, wave steepness, and so on. There is no agreement on the parameterization of the Charnock

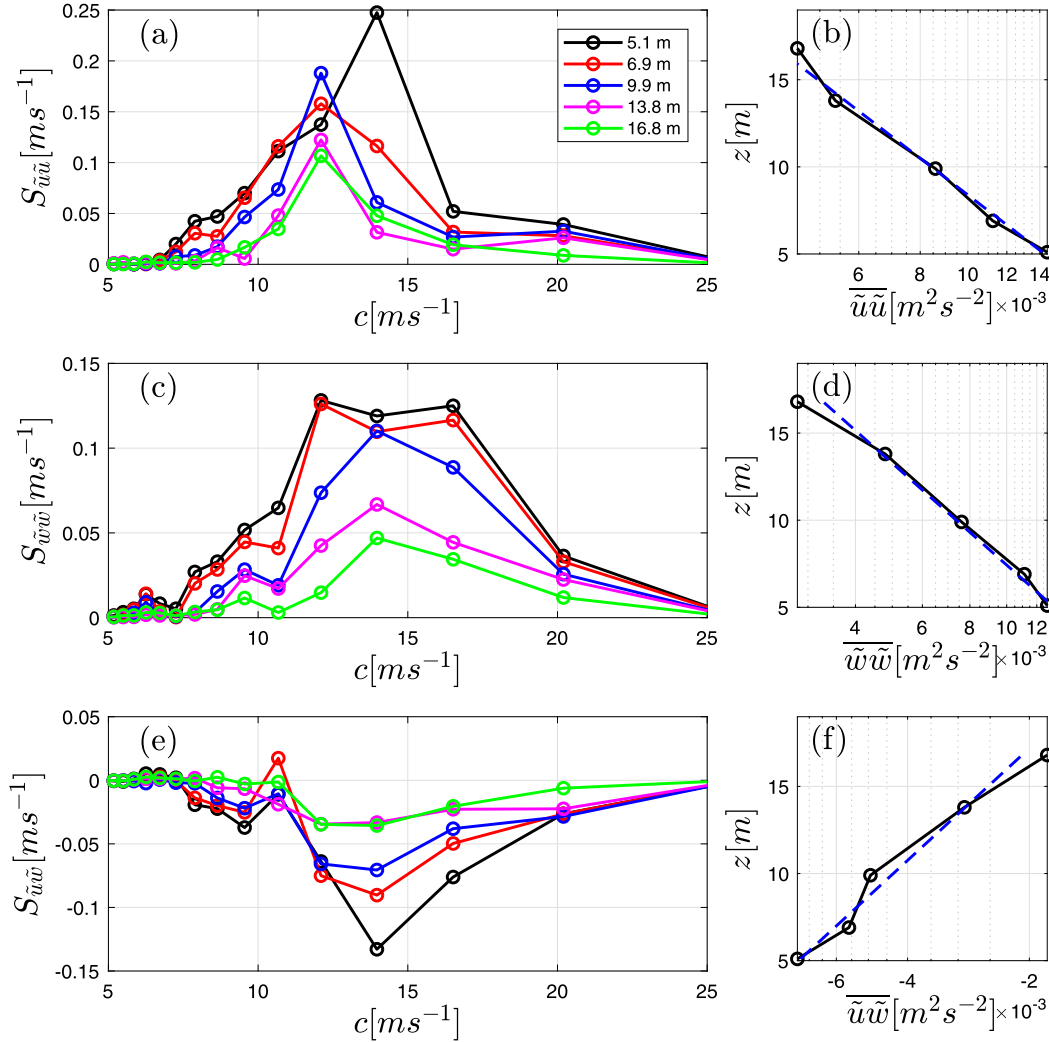


FIG. 10. The spectrum of (a) S_{uu} , (b) $\overline{uu}$, (c) S_{wv} , (d) $\overline{ww}$, (e) S_{uw} , and (f) $\overline{uw}$ at day 247.083; the dashed lines in (b),(d),(f) are the exponential fit lines.

number. Also, it is hard to calculate the value of Ω without the assumption of the logarithmic wind profile. Thus, we did not discuss the impact of Ω on the results. Here, we investigate the impact of k_p on the decay coefficients. To reduce the possible impact of atmospheric stability, only the data near neutral stratification are used, that is, $|z/L| < 0.1$. To reduce the impact of such data on the statistical results, only the cases in which the SIWC variances/momentum flux decay with height and in which the regression coefficient $R^2 > 0.7$ are used in the following statistical analyses.

The change in decay coefficients with the peak wavenumber is shown in Fig. 11. In general, the decay coefficients A_u , A_w , and A_{uw} decrease with increasing peak wavenumber k_p . The value of the decay coefficients is mainly in the 0.5–4 range. The solid lines

in Fig. 11 are the exponential fit lines, and the dashed lines represent the 95% confidence intervals. Compared with the decay coefficients A_u and A_{uw} , the decrease in A_w with increasing k_p agrees better with the exponential decay function. For A_u and A_{uw} , the data from RED are scattered across the exponential fit lines. However, the trend of the change in decay coefficients with k_p is significant, representing an approximately exponential decay with increasing values of k_p . The exponential fit lines of the decay coefficients A_u , A_w , and A_{uw} are $A_u = 3.27 e^{-16.8k_p}$, $A_w = 4.81 e^{-16.01k_p}$, and $A_{uw} = 4.87 e^{-17.75k_p}$, respectively. Similar to the theoretical results (Fig. 2), the decay coefficient A_{uw} is more sensitive to k_p than are A_u and A_w . However, the decay coefficient A_w changes more quickly with increasing k_p than does A_u , which differs from the theoretical

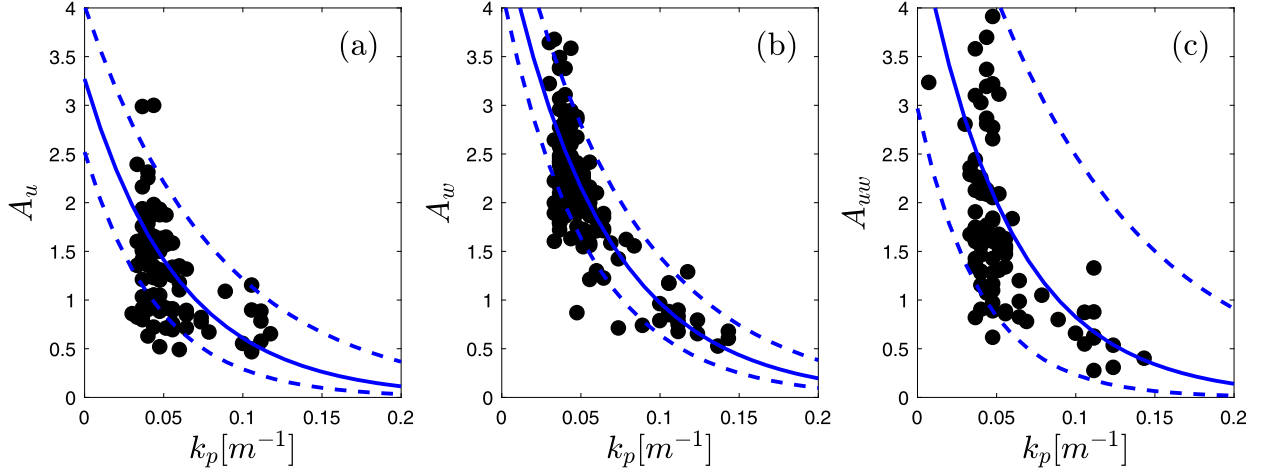


FIG. 11. The change in decay coefficients A_u , A_w , A_{uw} with peak wavenumber k_p for the RED data; the solid blue lines are the exponential fit lines, and the dashed blue lines indicate the 95% confidence intervals.

model results. One possible reason is that the wind profile difference can directly impact the results of $\tilde{u}$ and $\tilde{w}$. And the wind profiles in the open ocean may differ from the logarithmic wind profile used in the theoretical model since the logarithmic wind profile can be affected by waves, atmospheric stratification, and so on.

4. Wave impact on the wind profile

Based on the model results, the wave impact on the wind profile is investigated using a simple surface model in this section. Neglecting the viscous stress (which is only important for the first few surface centimeters of the atmospheric boundary layer), the total momentum flux τ over waves consists of two terms: $\tau = \tau_t + \tau_w$, where τ_t is the turbulent momentum flux, and τ_w is the WC momentum flux. Assuming the total momentum flux to be constant near the surface layer (which is roughly valid for RED data; see Fig. 2c in Höglström et al. 2013), the turbulent momentum flux can be parameterized through mean wind gradients; therefore,

$$\frac{dU}{dz} = \frac{\tau_t}{\rho_a K_m} = \frac{\tau - \tau_w}{\rho_a K_m}, \quad (19)$$

where ρ_a is the air density, and K_m is the turbulent eddy viscosity, which is usually parameterized by $K_m = \kappa z u_*$ or $K_m = \kappa z \sqrt{\text{TKE}}$ under neutral stratification conditions. Using $K_m = \kappa z u_*$ and integrating Eq. (19) produces

$$U(z) = \frac{u_*}{\kappa} \ln(z) - \frac{T_w(z)}{u_* \kappa} + C, \quad (20)$$

where

$$T_w(z) = \int_z^\infty \frac{\tau_w}{\rho_a z} dz = - \int_z^\infty \frac{\tau_w}{\rho_a z} dz = - \frac{\tau_{w0}}{\rho_a} \int_z^\infty \frac{e^{-A_{uw} k_p z}}{z} dz. \quad (21)$$

It is very reasonable to expect that the influence of the waves vanishes with height. Both the solutions of the Rayleigh equation as well as comparison of signals from instruments at different heights suggest exactly that $T_w(z) \rightarrow 0$ when z increases. In the real atmosphere, the logarithmic layer extends to a certain height, and the wave effects appear to be practically absent at such heights. Assuming that the wind profile over waves at some large height converges to a logarithmic shape $U(z) \xrightarrow{z \rightarrow \infty} (u_*/\kappa) \ln(z/z_0)$. Then, we can get $C = -(u_*/\kappa) \ln(z_0)$. Therefore,

$$U(z) = \frac{u_*}{\kappa} \ln(z/z_0) - \frac{T_w(z)}{u_* \kappa}. \quad (22)$$

We use the logarithmic wind profile and the wave-induced wind profile to represent the first and second terms in Eq. (22):

$$U_{\log}(z) = \frac{u_*}{\kappa} \ln(z/z_0), \quad \text{and} \quad (23)$$

$$U_w(z) = \frac{\tau_{w0}}{\rho_a \kappa u_*} \int_z^\infty e^{-A_{uw} k_p z} / z dz. \quad (24)$$

The ratio between the wave-induced wind and the logarithmic wind profile is therefore

$$R(z) = \frac{U_w(z)}{U_{\log}(z)} = \frac{\tau_{w0}}{\tau} \frac{\int_z^\infty e^{-A_{uw} k_p z} / z dz}{\ln(z/z_0)}. \quad (25)$$

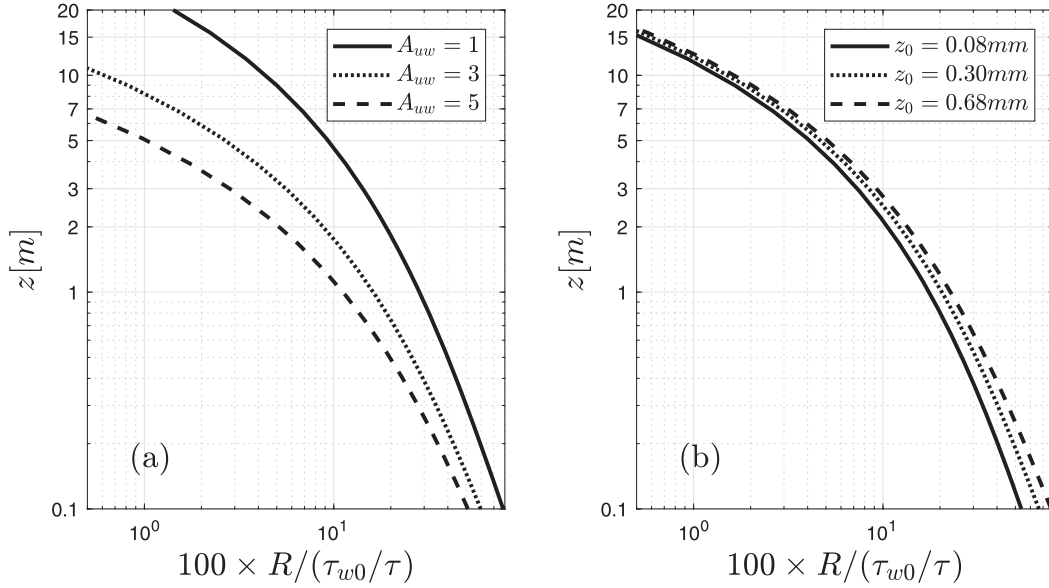


FIG. 12. The normalized ratio between the wave-induced wind and the original logarithmic wind profile for different values of (a) the decay coefficient A_{uw} with $z_0 = 0.30$ mm and (b) the surface roughness length with $A_{uw} = 2$.

The normalized value of $R(z)$ for several values of A_{uw} and the roughness length when $k_p = 0.06 \text{ m}^{-1}$ are shown in Fig. 12. The roughness length in Fig. 12a is 0.3 mm and $A_{uw} = 2$ in Fig. 12b. Three factors, that is, the ratio between the surface WC momentum flux and the total momentum flux, the decay coefficient of the WC momentum flux A_{uw} , and the roughness length determine the departure of the wind profile from the original logarithmic wind profile.

In the case of $\tau_{w0}/\tau = 20\%$, $A_{uw} = 1$, and $z_0 = 0.3$ mm, for example, the wave-induced wind can be about 6% (1%) of the wind speed predicted from the logarithmic wind profile at 1 m (10 m), respectively. With increasing height, the impact of WC momentum on the original logarithmic wind profile decreases. If the decay coefficient A_{uw} is larger, the impact of the WC momentum flux on the wind profile decreases rapidly with height, with the main impact being in the surface layer. The impact of z_0 on the wind profile is shown in Fig. 12b. The greater the roughness length z_0 , the greater the impact on the wind profile near the surface layer. The value of z_0 does not significantly change the influence of waves on the wind profile at height $z > 1$ m. The decay coefficient of the SIWC momentum flux therefore has a larger impact on the wind profile than does the surface roughness length. The results in section 3a(2) indicate that the ratio between the WC momentum flux and the total momentum flux is smaller under wind-wave conditions than under swell conditions (see Fig. 6). According to the results presented in Fig. 12, we can conclude that the impact of the SIWC momentum flux on the logarithmic wind profile is

smaller under wind-wave than that under swell conditions. The results agree with the observation showing that the logarithmic wind profile may be not valid under swell conditions (e.g., Semedo et al. 2009).

5. Discussion

The profiles of the SIWC momentum flux and velocity variances were investigated using a theoretical model and open-ocean measurements, mainly under swell-wave conditions. The results of the theoretical model using a logarithmic wind profile and measurements indicate that the SIWC momentum flux and velocity variances decay approximately exponentially with height. In the following subsections, certain aspects of these results are further discussed and possible applications to numerical models are introduced and discussed.

a. Impact of wind profile on wave-coherent momentum flux

An alternative expression for the WC momentum flux [Eq. (14)] in the theoretical model is (Miles 1959)

$$-\tilde{u}\tilde{w}(z; \chi_0) = \begin{cases} \frac{\pi}{2} \left(\frac{u_*}{\kappa} \right)^2 \frac{U_c''}{U_c'} k |\phi_c|^2 a^2 & \text{for } z_0 \leq z \leq z_c \\ 0 & \text{for } z_c < z \end{cases}, \quad (26)$$

where U_c'' and U_c' are the derivatives of the wind profile with respect to z at a height of z_c . From Eq. (26), one can

see that the WC momentum flux is very sensitive to U_c''/U_c' and ϕ_c . Thus, a small change in the wind profile may significantly change the magnitude of the WC momentum flux. In the theoretical study (section 2), the logarithmic wind profile [Eq. (15)] is used to calculate the WC momentum flux and velocity variances. However, in the open ocean, the wind profile can depart from its original logarithmic wind profile because of the influences of waves, atmospheric convection, and other factors. The SIWC momentum flux and velocity variances calculated from open sea measurements may therefore differ in some respects from the theoretical model results based on the logarithmic wind profile. This is one possible reason why the decay coefficients obtained from measurements are smaller than those from the theoretical model (see Figs. 2 and 11). Also the noises in the measurements and the wind-wave angle differences can also contribute to the decay coefficient difference between measurements and theoretical model. However, like the measurements, the theoretical model can offer a good understanding of the details of WC momentum flux in the layer near the surface, which displays behavior qualitatively similar to that captured by the measurements.

b. Wind-wave direction impact on the phase shift

Under wind-following/wind-crossing ($|\alpha_1| < 90^\circ$) wave conditions, the phase of $\tilde{u}$ is approximately 180° below the critical layer. This agrees with the results of Soloviev and Kudryavtsev (2010) and Grare et al. (2013). The phase of $\tilde{u}$ is approximately 0° under wind-opposing wave conditions, as has been demonstrated by Soloviev and Kudryavtsev (2010, see their Fig. 4). This may explain why the phase of $\tilde{u}$ for high wave speed modes is around 0° on days 246–254 since there is another swell system. The direction difference between this swell system and the wind direction is close to or larger than 90° . Also the signal-to-noise ratio is low under the high wave speed modes, and the resolution of the directional wave spectra is only 90° for the measurements.

c. Wave-coherent upward momentum flux

During most days of the RED experiment, the WC momentum flux was downward (i.e., from atmosphere to ocean) for all wave modes. However, the WC momentum flux for the high-phase speed wave modes was upward (i.e., from ocean to atmosphere) in some situations on days 256.5–257. It is worth noting that the wind speed during this period was very low and the wave age was high. This agrees with previous studies (Grachev and Fairall 2001; Grare et al. 2013) finding that the total momentum flux can be upward under swell and light

wind conditions. Makin (2008) demonstrated that upward momentum flux can be caused by long waves under light winds (i.e., $U < 2 \text{ m s}^{-1}$).

To our knowledge, few studies have considered physical explanations for the transition between the downward and upward momentum fluxes. One possible explanation is that the wave-induced Stokes drift in the air may occur under very low wind conditions (Harris 1966), which is associated with the wave to air momentum flux (Hristov and Ruiz-Plancarte 2014). Under such conditions (i.e., very low winds), the shear-induced turbulence is virtually absent and the atmospheric motion is driven by the Stokes drift and buoyancy (Hristov and Ruiz-Plancarte 2014), with the Stokes drift supporting the upward momentum flux, as has been indicated by measurements. In addition, in the large-eddy simulations of Sullivan et al. (2014), the positive cospectrum contribution of small wavenumbers was observed under high wave age conditions. The upward momentum flux is therefore supported by those wave modes. An alternative explanation for the upward momentum flux being contributed by the Stokes drift is that the transition from downward to upward momentum flux is due to ripples (Högström et al. 2017, manuscript submitted to *J. Atmos. Sci.*). Under light winds and swell conditions, the sea surface is dynamically smooth, and the swell-induced upward form drag dominates the WC momentum flux. With increasing wind, ripples emerging at the sea surface increasingly contribute to the downward momentum flux. However, there is still no general agreement on criteria and explanations concerning the transition from downward to upward momentum flux. It is clear that the transitions from upward and downward momentum fluxes depend on both wind speed and wave conditions. Further studies are needed to explore the physical explanation and criteria for the upward momentum flux over the ocean.

d. Wave-coherent pressure fluctuations

Based on the numerical solution of Eq. (5) and the logarithmic wind profile, the WC pressure can be given as

$$\tilde{p} = \rho_a \left(\frac{u_*}{\kappa} \right)^2 \left(\chi \frac{d\phi}{d\xi} - \xi \frac{d\chi}{d\xi} \right) k\eta. \quad (27)$$

Figure 13a shows the WC pressure $\tilde{p}$ at different heights for the case with $u_* = 0.4 \text{ m s}^{-1}$ and several spectrum wave ages, that is, c/u_* . For the wave modes with $c/u_* = 10$ and 15, the critical layer height is less than 1 m. The critical height for the $c/u_* = 26$ case is approximately 14 m with an abrupt change in $\tilde{p}$ close to the critical height. However, the critical layer height for the

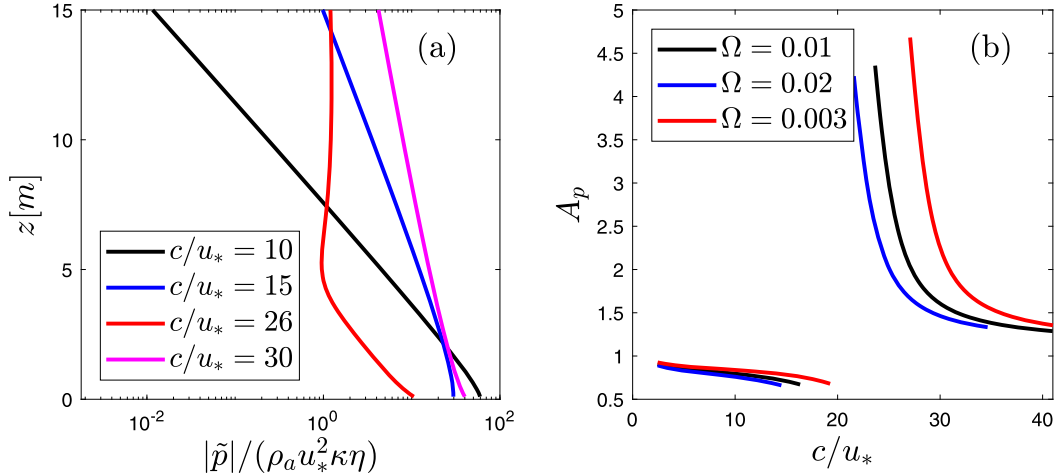


FIG. 13. (a) The wave-coherent pressure decay with height for different values of c/u_* at $\Omega = 0.003$ and $u_* = 0.4 \text{ m s}^{-1}$, and (b) the decay coefficient at different wave ages for different values of Ω with $u_* = 0.4 \text{ m s}^{-1}$.

$c/u_* = 30$ wave mode is about 64 m. The WC pressure $\tilde{p}$ decays approximately exponentially with height (Fig. 13a) above/below the critical layer (there is an abrupt change around the critical layer height). Using the simulation results for heights of 0–15 m, the exponential decay coefficient A_p in $\tilde{p}(z) = \tilde{p}(0)e^{-A_p k z}$ is shown in Fig. 13b. Only the cases with the regression coefficient $R^2 > 0.98$ are shown in Fig. 13b. For cases with a low wave age c/u_* the decay coefficient is slightly less than 1. However, the decay coefficient decays with c/u_* in high wave age cases. In very high wave age cases, the decay coefficient A_p is approximately 1.3. For the wave modes with a critical layer height in the commonly measured range, $\tilde{p}$ does not exponentially decay with height (see the gap in Fig. 13b) since the value of $\tilde{p}$ has an abrupt change around the critical layer height [see also Fig. 4a in Hristov (2017)]. The dimensionless parameter Ω can also affect the results (see the differently colored lines in Fig. 13b) since it can affect the wind profile used in Eq. (27).

In a recent study, Kahma et al. (2016) used the exponential decay function to extrapolate the WC pressure at the surface from a measured height to calculate the wave growth ratio. Their theoretical model results may overestimate/underestimate the wave growth rate if the decay coefficient is assumed to be 1. One should therefore be careful when extrapolating the WC pressure at a measured height to the surface.

e. Possible applications to numerical models

From the theoretical model and the RED experiment measurements, the SIWC momentum flux and velocity variances were found to decay approximately

exponentially with height, and the decay coefficients were found to decrease with increasing peak wavenumber. In the RED experiment data, the scatter of the decay coefficients is significant, especially in cases with low peak wavenumbers. This is an indication that other factors may affect the decay coefficients, such as the Charnock coefficient, wave spectrum exponent β , and wind profile. We investigated the other factors correlated with the decay coefficients but found no significant relationship in the RED data. The idealized large-eddy simulation (LES) cases with wind-following swell and low wind conditions show that the decay coefficient for the pressure stress (i.e., the dominant term of the WC momentum) decays with increasing wavelength, wave slope, and wave age (Jiang et al. 2016). However, the theoretical model and RED data results differ from the idealized LES results. One possible reason for this difference is that the LES cases used by Jiang et al. (2016) are cases involving WC upward pressure stress. Another possible reason is that the LES model used by Jiang et al. (2016) is an idealized model that ignores some other influential factors, such as the wave–wave interaction and wave spectrum influence. The noise in the RED data may also contribute to the differences.

In the study of Wu et al. (2016), the WC momentum flux is added to a regional atmosphere–wave coupled model though parameterizing the WC momentum flux into an effective roughness length based on the study of Höglström et al. (2015). Adding the swell impact into the effective roughness length improves the agreement between the model results and measurements. However, in that study, the model is still based on MOST and the impact of the WC momentum flux profile on the model is not introduced. As shown in Fig. 12, the wind profile is

still a logarithmic wind profile when changing the roughness length; this changing the roughness length cannot capture the wave influence on the wind profile [Eq. (15)]. After adding the profile of the WC momentum flux under swell conditions, the model can simulate the wave-induced low-level wind jets (Wu et al. 2017). In further model development, the profile of the WC momentum (not only the WC surface momentum flux) should therefore be included in atmospheric models.

6. Conclusions

The vertical profiles of the SIWC momentum flux and velocity variances are investigated based on a theoretical model and on measurements from R/P *FLIP*. The SIWC momentum flux and velocity variances determined from measurements are estimated based on the Hilbert transform at different heights. The results of the RED experiment measurements qualitatively agree with the theoretical model results.

The SIWC momentum flux and velocity variances decay approximately exponentially with height. In general, the exponential decay coefficients decay with increasing peak wavenumber (however, it displays significant scatter), which does not agree with a constant coefficient used in previous studies. The SIWC momentum flux decays more quickly with height than do the SIWC velocity variances. The SIWC momentum flux ratio becomes larger with decreasing height, exceeding 20% of the total momentum flux at a height of 10 m under swell and light wind conditions. Compared with $\overline{u^2}/u^2$, $\overline{w^2}/w^2$ is larger at the same height.

The impact of the WC momentum flux on the logarithmic wind profile is determined by the ratio between the surface WC momentum flux and the total surface momentum flux, the decay coefficient of the WC momentum flux, and the roughness length. The ratio between the surface WC momentum flux and the total surface momentum flux is the dominant one of these three factors.

The phases of the WC vertical and horizontal velocities are approximately 90° and 180° below the critical height under wind-following wave conditions, respectively. The scaled amplitude of the wave-induced velocities increases with wave age for fast waves, that is, $c/U > 1$; though, they stop increasing when the wave age is $c/U \approx 3$.

The total wind stress over ocean was parameterized through two separate terms, that is, the WC momentum flux and the residual momentum flux (Högström et al. 2015). The results of this study illustrate how to parameterize the surface WC momentum flux based on the exponential decay estimation with parameterized decay

coefficient. This will lead to a new way to parameterize the total momentum flux over ocean.

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